

# 11 - Gears

Although motors are the best way to drive most mechanisms, they are fairly useless without gears to go along with them. The problem with motors alone is that they are usually far too fast to be useful. Without any load on them, even the slowest FRC motors spin at over 5,000 revolutions per minute (rpm), and the fastest can reach over 18,000 rpm. If a 5,000-rpm motor were to directly drive a four-inch drive wheel, the robot would move at over 125 miles per hour! That is, if it would move at all, since motors deliver very little torque when they spin that fast. Therefore, gears are needed to reduce the speed and increase the torque of spinning parts to useful levels.

## 11.1 - Gear Ratios

Gears are mechanical components which cause rotation in each other by pushing on each other with physical contact. Many types exist, but they all use interlocking teeth to spin together. This guide will focus mostly on spur gears, the most common type, which have straight teeth and rotate in axes parallel to each other.



Figure 1: Two interlocking spur gears. Note how the teeth lock together  
(Image from [MotionControlTips.com](http://MotionControlTips.com))

These interlocking teeth are the basis of gear motion. Since the teeth lock together, the edges of the gears must move at the same speed and in the same direction. Figure 2, below, illustrates this, with the gears simplified as circles.

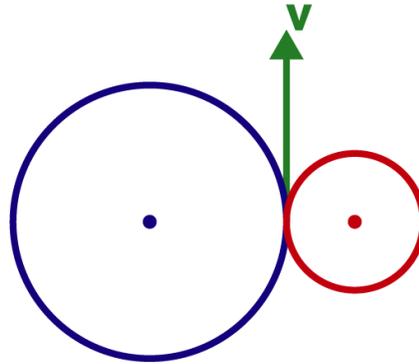


Figure 2: Diagram showing that the velocity of interlocked gear surfaces are equal

When we work with gears, however, we're most interested in how fast they spin, not how fast they move. So, if we know the radii of the gears, we can find their rotational (also called angular) velocities. In the diagram, we'll label the radii  $r_1$  and  $r_2$ , as shown. We'll also label their angular velocities with the Greek letter lowercase omega,  $\omega_1$  and  $\omega_2$ .

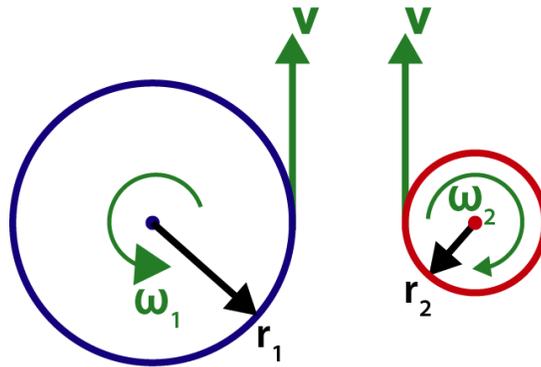


Figure 3: Fully labelled diagram

Now that the parameters have been defined, we can apply the relationship between linear and angular velocity, which is given by Equation (1).

$$v = \omega r \quad (1)$$

Seeing that the linear velocities are equal, we can relate the radii and angular velocities by Equation (2)

$$\omega_1 r_1 = \omega_2 r_2 \quad (2)$$

Thus, we can define the **gear ratio**, the ratio between the speeds of the two gears, by Equation (3).

$$R = \frac{\omega_2}{\omega_1} = \frac{r_1}{r_2} \quad (3)$$

As you'll see going forward, the gear ratio is arguably the most important factor in the design of gear systems, since it allows us to calculate many different design parameters.

## 11.2 - Gear Pitch

Gears come in many different sizes, with varying diameters, thicknesses, and tooth sizes, and gears with different tooth sizes will not mesh together. They can be classified, among other things, based on their pitch. In general, a **pitch** is the distance between each tooth or link in a component, and it applies to things like sprockets and belts as well. For example, if a belt has one tooth every five millimeters, we say that it has a 5-mm pitch. This concept can apply to gears, too, but because gears are circular it is more convenient to define the pitch of a gear based on its diameter. Therefore, the **diametral pitch** of a gear is the number of teeth around the circumference per inch of diameter. Most gears used in FRC are 20-DP gears, meaning that they have a diametral pitch of 20 teeth per inch. The number of teeth in a gear can be calculated based on its diameter, according to Equation (4). Here,  $N$  is the number of teeth,  $D$  is the pitch diameter, and  $DP$  is the diametral pitch.

$$N = D \cdot DP \quad (4)$$

We can rearrange Equation (4) and see that the diameter of a gear can be derived from its number of teeth and its pitch, as in Equation (5).

$$D = \frac{N}{DP} \quad (5)$$

So, if we want to know the gear ratio between two gears, we can calculate that based on the number of teeth. Recalling that the radius is half the diameter, the gear ratio definition in Equation (3) becomes Equation (6).

$$R = \frac{D_1}{D_2} = \frac{\frac{N_1}{DP}}{\frac{N_2}{DP}} \quad (6)$$

Finally, since the pitch must be the same for two interlocking gears, we can simplify this to Equation (7).

$$R = \frac{N_1}{N_2} \tag{7}$$

Thus, we have an equation for the gear ratio based on the size of the gears and their number of teeth.

## 11.3 - Center-to-Center Distances

In addition to the gear ratio, we can also extract the center-to-center (C-C) distance between two gears from their pitches and tooth counts. Equation (8) relates the number of gear teeth for two gears,  $N_1$  and  $N_2$ , and their pitch diameter to the distance between their centers,  $C$ . Figure 4 illustrates this, too.

$$C = \frac{D_1 + D_2}{2} = \frac{N_1 + N_2}{2DP} \tag{8}$$

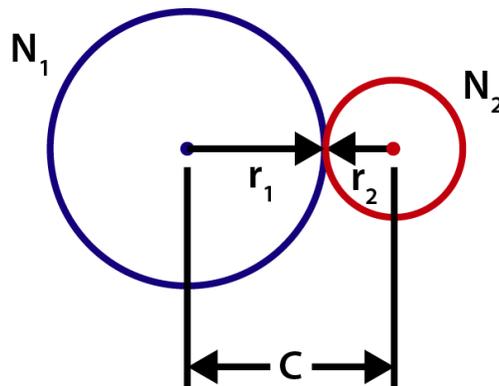


Figure 4: Diagram showing the center-to-center distance of two gears

One more consideration, however, is that gears have multiple diameters that we designers need to keep track of, as illustrated by Figure 5. The diameter in Equations (4) and (5), as well as the radii used in Section 11.1, are the **pitch diameter** (and pitch radii), which is the diameter on which gear spacing, velocities, and other design factors are calculated. It is roughly the diameter of the center of the tooth face. The **outer diameter**, on the other hand, is the diameter of the tips of the gear teeth. It is not used to calculate gear spacing or anything of that sort, but it is important to consider because, clearly, it would be a problem if gear teeth collided with other components.

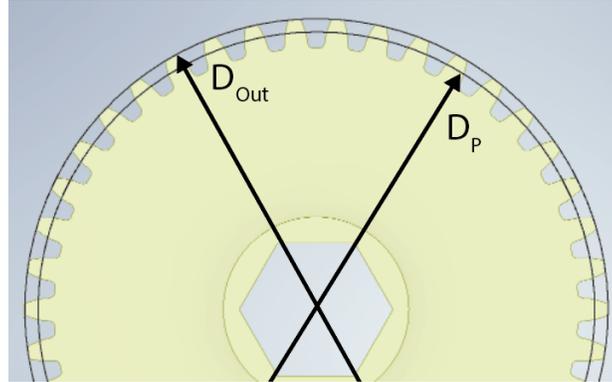
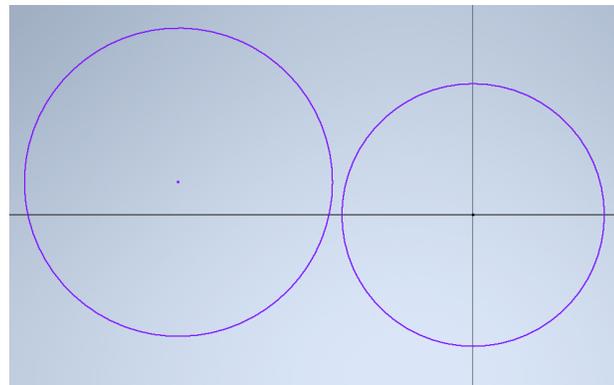


Figure 5: Pitch diameter,  $D_p$ , and outer diameter,  $D_{out}$ , of a gear

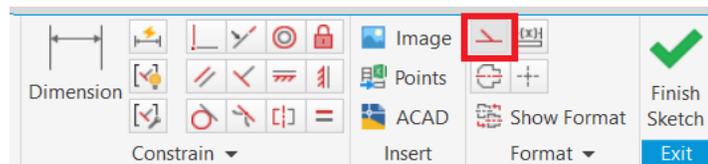
## 11.4 - Tutorial: Gear C-C Distances in Inventor

Although it is important to understand the calculations behind gear geometry, taking advantage of geometric relationships in Inventor makes designing with gears quite simple. For this tutorial, we will simply attach two gears to a flat plate.

*Start by creating a sketch on the front plane and placing two circles anywhere in it. Place the first with its center at the origin, but do not constrain the second one yet.*

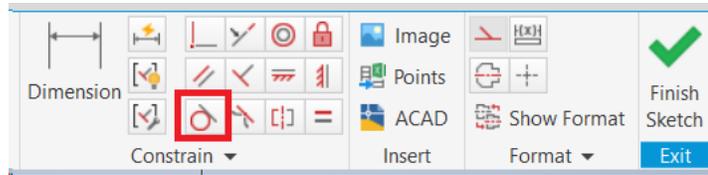


*Select the circles, holding CTRL to select both, and then choose "Construction" from the top-right corner of the ribbon.*

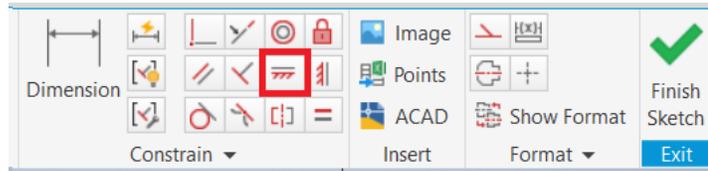


Construction geometry is any lines and curves which do not contribute to the closed geometry of the sketch, so it has no effect on any extrusions, revolutions, or other features based on the sketch. It is displayed as a thin dashed line. Next, we will apply the geometric relationships necessary for these circles to model gears.

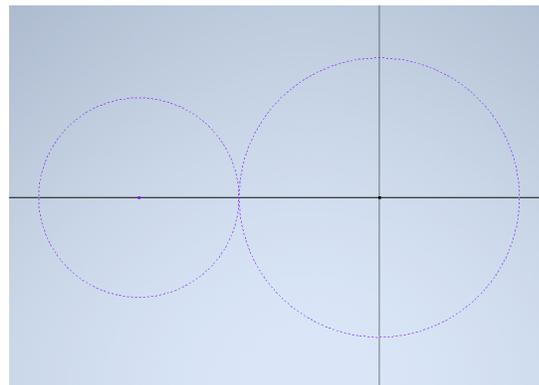
With the circles still selected, click on the Tangent constraint to apply it. The circles will move into contact with each other.



Select the two centers and add a horizontal constraint.



Now, try dragging the center of the non-origin circle back and forth. Note that when centers of the two circles move farther apart the diameter of the moving circle increases.

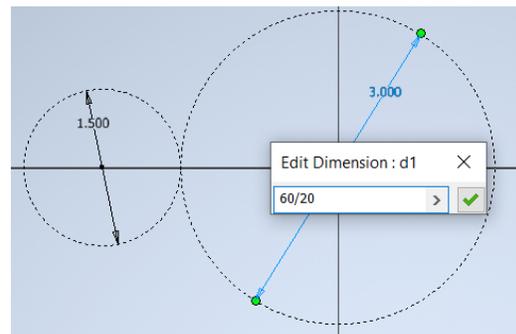


The way that the diameter increases with the center-to-center (C-C) distance is expected based on the geometric relationships explored in Section 11.2. It turns out, in fact, that we only need to define the tangent constraint between the circles and the diameters of the circles for Inventor to accurately calculate the C-C distance. But, we know the relationship between the number of teeth, the diametral pitch, and the diameter, so we can exploit that relationship to make our modelling easier. We will use equations inside dimensions to do this.

The number of teeth of the gears depends on the mechanism and is a result of calculations that are beyond the scope of this design guide, so you can expect to be given the tooth counts of the gears. For this case, let's say that we were told we need a 60-tooth gear and a 30-tooth gear, both 20 DP.

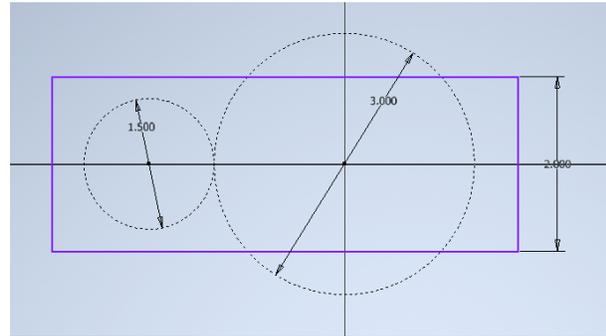
Apply dimensions to both circles. For the first, type "60/20", and for the second, "30/20".

This essentially is Equation (4), and you could calculate the diameters by hand before applying the dimensions, but it's easiest to let Inventor handle the calculations.

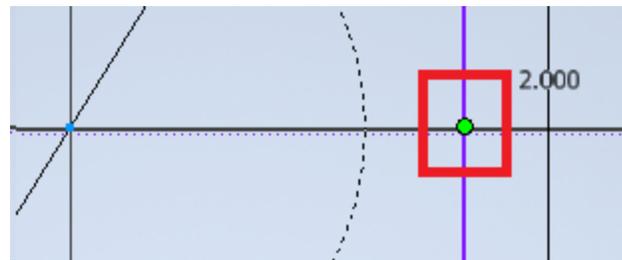


Now, we just need to design the rest of this plate. We'll use the construction geometry to create the geometry of the plate.

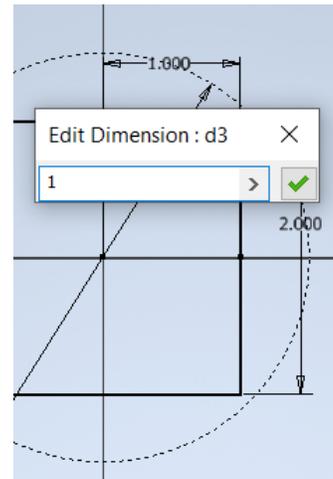
Create a rectangle around the centers of the circles, as shown. Set the height to 2", but do not specify the width yet.



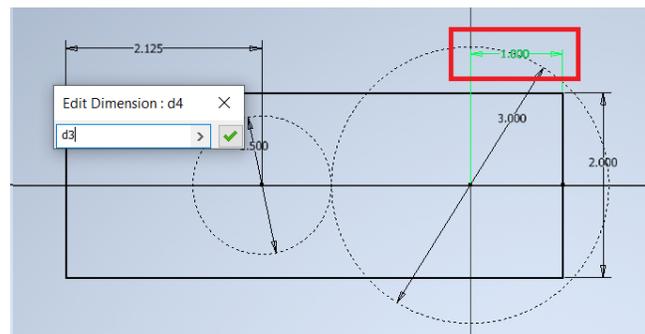
Select the Horizontal Constraint tool, then click on the origin and the midpoint of the right vertical line. The midpoint will turn green before you select it.



Then, define a 1" dimension between the origin and the same vertical line.

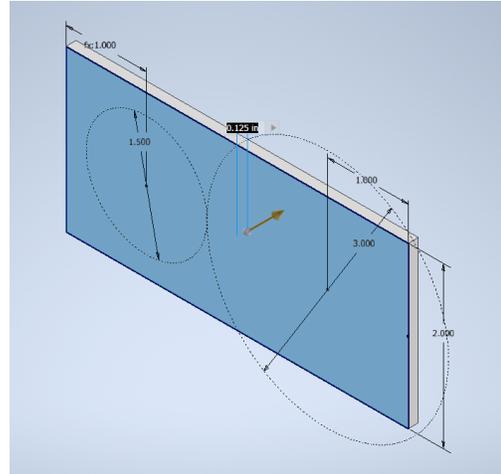


Finally, define another dimension between the other center point and the other vertical line. Link this new dimension to the last one by clicking on it. It will turn green before you select it.

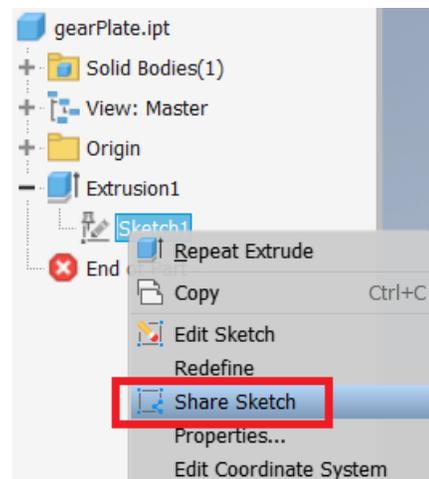


By linking the dimensions, they will always be equal. You can also use equations in this way, so you can define one dimension as, for instance, half of another one or the sum of two others. The sketch is now complete, so we will extrude the part. Then, we will share the sketch that we just used to define the hole centers.

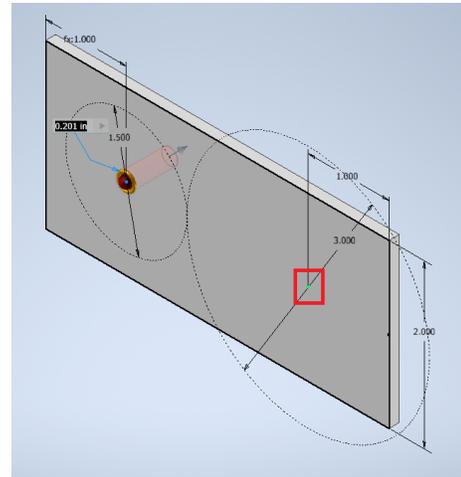
Create an extrusion with flipped direction and a 0.125" thickness.



In the Model Tree, select the plus sign by "Extrusion 1" to expand it, and then right-click on "Sketch 1". From the list, choose "Share Sketch". The sketch will now be visible again.

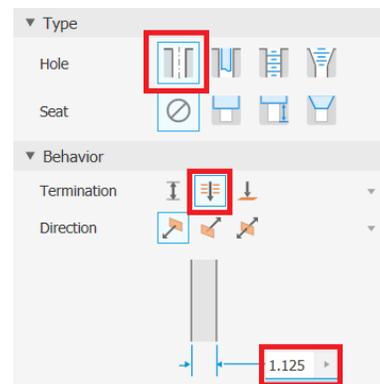


Now select the Hole tool from the ribbon and choose the two center points from the sketch.

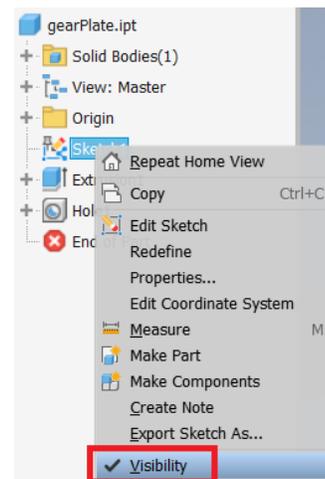


Set the hole type to “Simple Hole”, the termination to “Through All”, and the diameter to 1.125”. This is the diameter of an ordinary ½” hex bearing.

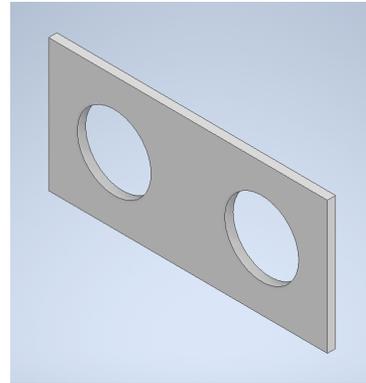
Select “OK” to complete the holes.



From the Model Tree again, right-click on “Sketch 1” and select “Visibility” to turn the sketch invisible again.



*The part is now complete.*



This is now a plate to which we can attach the gears which we specified before. After completing the assembly by adding shafts and bearings, this simple gearbox is shown in Figure 6.

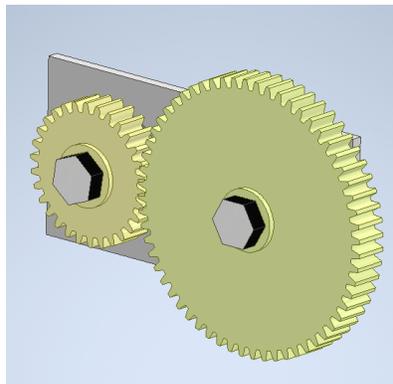


Figure 6: Semi-completed tutorial assembly

Obviously, this is not a truly complete assembly - the shafts are supported only on one end, it doesn't have shaft collars, it has no way to connect to other mechanisms, and so on - but hopefully it is a thorough demonstration of how to design with gears. True gearbox design will be explored in more detail later.

## 11.6 - Other Types of Gears

Sometimes, an ordinary spur gear does not fit the design requirements, such as if two shafts are oriented at an angle from each other or if a large reduction is needed in a small space. In these cases, other types of gears can be used. Table 1, below, lists several types and descriptions of them.

Table 1: Non-spur gears

| Image  | Type                  | Description  |
|--|-----------------------|--|
|  <p>(image from <a href="http://MechanicDrive.com">MechanicDrive.com</a>)</p>       | <p>Bevel Gear</p>     | <p>Bevel gears connect shafts which rotate perpendicular to each other using flat teeth turned inward at an angle.</p>   |
|  <p>(image from <a href="http://MSCDirect.com">MSCDirect.com</a>)</p>              | <p>Worm Gear</p>      | <p>Worm gears also connect shafts that are perpendicular to each other, but they use a small screw-like piece (the worm) that drives a circular gear (the worm wheel.) They offer a large gear ratio in a relatively small space. Because of the spiral on the worm, they are difficult to back-drive.</p> |
|  <p>(image from <a href="http://HMManufacturing.com">HMManufacturing.com</a>)</p> | <p>Planetary Gear</p> | <p>Planetary gears allow large gear ratios in a small package. They work by having a small sun gear in the center rotate planet gears around it, which are connected to the outermost ring gear. Typically, the ring gear is fixed, and the planet gears rotate a carriage.</p>                            |

## 11.5 - COTS Gears

When designing with gears, it is important, of course, to know which gears are available from COTS suppliers. We almost exclusively use gears sold by VexPro. They offer 20-DP spur gears in a wide range of sizes and in two-tooth increments, providing great degree of flexibility for design. They also sell 32-DP spur gears, which allow greater

gear ratios in tighter spaces, a small selection of bevel gears, and a very convenient customizable planetary gearbox system.



Figure 7: VexPro spur gear  
(image from [VexRobotics.com](http://VexRobotics.com))